Excel Solver Wizard Model Development for Abeokuta's Rainfall Intensity, Duration and Frequency

Abstract

In hydraulic engineering systems, the kind and speed of water flow that results from rainfall activity in a particular catchment are constant variables. Twenty-five (25) years' worth of Abeokuta daily rainfall data (amount and duration) were provided by the Nigerian Meteorological Agency (NIMET), Abuja. These data were then subjected to frequency analysis in order to create intensity-duration-frequency models. Mean rainfall levels for the durations of 5, 10, 15, 20, 30, 45, 60, 90, 120, 180, 240, 300, and 420 minutes were retrieved and subjected to frequency analysis using the Excel Optimization Solver wizard. Using the Normal distribution and Pearson type 3 distributions, defined and non-specified IDF models were developed for return periods of 2, 5, 10, 25, 50, and 100 years. Abeokuta has not witnessed these models' development. Values of the mean squared error (MSE) and coefficient of determination (R^2) were used to assess the probability distribution functions' fitness. The normal distribution has R^2 and MSE values between 0.977 and 0.991 and 85.73 and 118. 14, but the Pearson type 3 distribution has values between 0.964 and 0.997 and 42.88 and 118.68. It is recommended that the Ministry of Works utilize probability distribution models to anticipate rainfall intensities in the city of Abeokuta, in order to set acceptable design objectives.

Keywords: Excel Optimization Solver, Normal distribution, Pearson Type 3 distributions, Goodness of fit test, IDF models, Abeokuta.

I. Introduction

Effective management of water resources in all river basins can help fulfill the first and second Sustainable Development Goals of the UN, which are to end poverty and ensure food security (https://sdgs.un.org/goals). Statistical analysis approaches on rainfall volume and duration lead to the construction of Rainfall Intensity Duration Frequency connections that may be used for effective study and design of flood control systems. As per David et al. (2019), effective planning and design measures for extreme events such as floods, droughts, and rainstorms have a higher chance of success when there is a proper comprehension of the frequency of severe events. El-Sayed (2011) argued that projects involving water resources required a strong foundation in IDF modeling. Probability distribution functions are utilized for the frequency analysis of rainfall volume and duration from rain gauge stations. The IDF formulae, which are empirical equations, reflect other pertinent variables, which are independent variables, and the maximum intensity of rainfall as the dependent variable (for example rainfall length and frequency). Numerous of these probability functions are employed in real-world hydrological applications, according to Chow et al. (1988). Researchers and academics from all around the world have become interested in correct assessment of the intensity-duration-frequency relationship due to its widespread application (Mohammed Zakman, 2016). The IDF models in Port Harcourt show that coverage in Nigeria has spread from the North Central region to the South-East and South-South [Ilaboya & Nwachukwu, 2022; Nwaogazie & Masi, 2019) and Eket in the Awka Ibom State (Nwaogazie & Uba, 2001). For return periods of two to ten years, these results validate the IDF theory. With sufficient knowledge of the IDF, climate smart agriculture methods can be put into reality. The removal of barriers faced by small-scale farmers, such as lack of access to technical skills, inadequate market access, and inadequate investment, depends on the management of natural resources (land and water) (Morton, 2007).

The creation of these models is essential to the effective building of structures designed to mitigate flooding in Abeokuta. The last flooding disaster occurred in 2022, resulting in several million Naira in property losses and fatalities.

II. Materials and Methods

A. Study Area Description

The capital of Ogun State, located in southwest Nigeria, is Abeokuta. Situated among a cluster of rocky outcrops in a forested savanna, on the east bank of the Ogun River, it is approximately 130 kilometers by sea or 77 kilometers north of Lagos by train and covering an estimated metropolitan area of about 879 km^2. The elevation is 64 m above the sea level and falls within latitude 07°09’20”

N and longitudes 03°20’42” E. The study area is graphically illustrated in Figure 1.

B. Data Collection

The Nigerian Meteorological Agency (NIMET) provided the data, which spanned the 25-year period from 1986 to 2010. ‘Five to four hundred- and twenty-minutes’ worth of data were separated into intervals. Rainfall intensities might be computed for the construction of different models by using the ranking data.

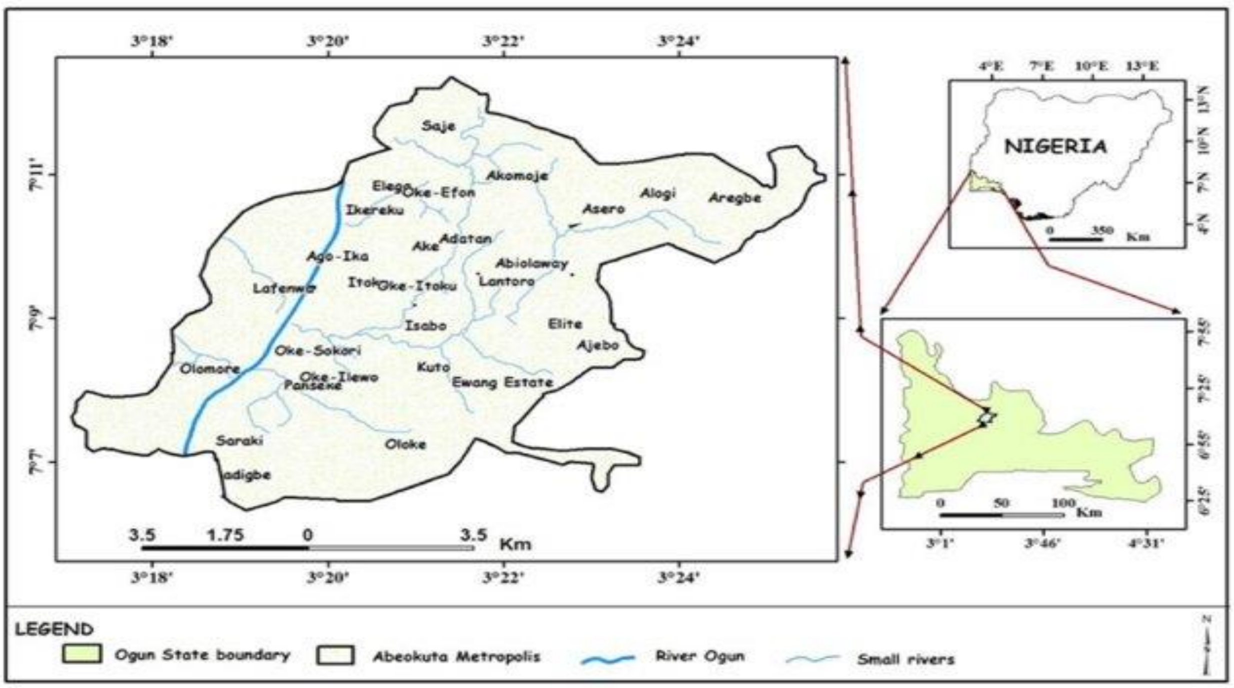


Figure 1: Map of Ogun State showing Abeokuta (Google., 2024)

C. Data Analysis

The durations of 5, 10, 15, 20, 30, 45, 60, 90, 120, 180, 240, 300, and 420 (minutes) were selected as having the highest rainfall. Equation (1) shows the mathematical formulation of the IDF connection as defined by (Chen, 1983; David et al., 2019).

:

$$I = f(T,d) \qquad (1)$$

Where: I = intensity; T = return period and d = duration.

Table 1 displays all of Abeokuta's intensities for various time frames.

A frequency analysis method was used to assess the rainfall intensity magnitude for the station under consideration. Since Pearson Type III and the Normal distribution best fit existing rainfall models, these two probability distribution functions have been adopted for computing rainfall intensities for specific return periods, according to an earlier work by Nwaogazie & Masi (2019).

D. **Normal Distribution**

The normal distribution is a frequently used probability distribution for estimating rainfall intensity values. The following equation (2) was utilized to get the rainfall intensity numbers:

$$X_T = X + K_T S \qquad (2)$$

Where X_T = rainfall intensity values (magnitude of the hydrologic event)

Table 1: Ranked Observed Annual Rainfall Intensities (mm/hr) for different Durations (mins) for Abeokuta

	Intensity (mm/hr) versus Duration (Minutes)												
Year	5	10	15	20	30	45	60	90	120	180	240	300	420
1	421.2	271.2	217.2	186.3	140.6	112.4	88.6	59.8	54.2	40.9	32.1	25.7	18.3
2	381.6	270.0	189.6	174.3	129.6	93.7	84.3	59.5	44.7	36.1	30.7	24.6	17.5
3	336.0	257.4	180.8	166.8	129.2	89.6	82.3	59.1	44.3	30.7	27.1	21.7	15.5
4	330.0	248.4	180.0	162.9	125.6	86.5	70.3	58.7	44.1	29.8	25.6	20.6	14.7
5	295.2	231.0	178.8	142.2	124.2	86.4	67.2	54.9	41.2	29.5	23.1	20.5	14.6
6	289.2	221.4	171.6	135.6	116.2	86.1	64.9	54.7	41.0	27.4	22.3	18.4	13.2

7	233.1	210.6	169.2	135.0	94.8	85.5	64.8	44.8	33.6	27.3	22.2	17.9	12.8
8	223.1	190.8	167.2	134.1	90.4	85.3	64.6	43.9	33.0	22.4	20.6	17.7	12.7
9	196.8	171.0	165.6	128.7	89.4	84.9	64.0	43.2	32.4	22.0	20.5	16.5	12.2
10	195.6	168.0	154.0	126.9	85.8	82.8	63.7	43.1	32.3	21.6	19.7	16.4	11.8
11	187.2	165.0	147.6	125.4	83.6	78.3	62.1	42.7	32.0	21.5	17.7	15.2	11.7
12	186.1	152.4	140.4	124.2	82.8	77.6	58.7	42.5	31.9	21.4	16.9	14.9	11.6
13	181.2	147.6	131.2	123.0	82.0	63.2	58.2	39.1	29.4	21.3	16.5	14.6	10.8
14	170.4	146.9	127.2	122.4	81.6	60.3	47.4	38.8	29.1	21.2	16.2	13.2	10.7
15	167.5	144.6	120.4	115.5	77.0	57.2	44.5	37.1	28.0	20.5	16.2	13.0	10.3
16	162.3	140.6	112.1	110.7	73.8	55.7	44.1	35.9	27.8	19.6	16.0	12.9	9.8
17	161.0	124.0	112.0	95.4	70.6	55.2	42.9	33.9	26.9	19.6	15.9	12.8	9.7
18	149.5	117.9	107.3	92.5	67.6	54.7	42.6	32.9	26.8	19.4	15.6	12.8	9.4
19	149.0	117.2	96.4	90.3	63.6	53.9	41.8	32.5	24.7	18.1	14.9	12.7	9.3
20	137.9	111.6	94.6	88.6	60.2	51.6	41.0	31.6	23.7	17.9	14.7	12.5	9.2
21	135.6	105.5	90.0	78.1	59.6	51.3	38.5	29.1	23.7	17.2	14.7	12.3	9.1
22	119.7	102.2	89.5	74.3	56.7	45.5	38.0	28.7	22.5	17.1	14.6	12.2	8.9
23	117.7	101.4	80.5	73.8	56.4	43.3	37.5	28.6	22.4	16.5	14.2	12.1	8.8
24	117.4	98.4	78.0	72.3	50.7	43.0	35.7	28.4	21.8	15.8	14.1	11.7	8.7
25	115.8	98.4	77.4	66.5	49.1	41.3	35.5	27.3	21.5	15.5	13.5	11.7	8.5
Mean	206.4	164.5	135.1	117.8	85.6	69.0	55.3	41.2	31.7	22.8	19.0	15.8	11.6
Standard Deviation	86.8	57.5	40.7	33.6	27.3	19.7	16.1	10.9	8.7	6.5	5.3	4.1	2.8
Coefficient of Skewness	1.05	0.63	0.16	0.25	0.64	0.26	0.51	0.53	0.95	1.32	1.19	1.08	1.01

The model parameters C, m, and a, were obtained by calibrating the non-linear power law given by Equation (4) using the Excel optimization solver.

$$K_T = w - \frac{2.515517+0.802853w+0.010328w^2}{1+1.432788w+0.189269w^2+0.001308w^2} \qquad (3)$$

where w = Intermediate Variable and is given in Equation(3a) as:

$$w = \left[In\left(\frac{1}{P^2}\right)\right]^{1/2} \qquad (3a)$$

and P = exceedance probability given in Equation (3b) as:

$$P = \frac{1}{T} \qquad (3b)$$

where T = return period

Example: Normal distribution frequency factor for 5 years return period

P = $\frac{1}{5}$ = 0.2

w = $\left[In\left(\frac{1}{0.2^2}\right)\right]^{1/2}$ = 1.794

Substituting our values into Equation (3) we have;

$$K_T = w - \frac{2.515517+0.802853(1.794)+0.010328(1.794)^2}{1+1.432788(1.794)+0.189269(1.794)^2+0.001308(1.794)^3} = 0.841457$$

Table 2 shows K_T values for Normal distribution for different return periods as calculated.

Table 2 Normal distribution frequency factor

Return Period	2	5	10	25	50	100
P	0.5	0.2	0.1	0.04	0.02	0.01
W	1.17741	1.794123	2.145966	2.537272	2.79715	3.034854
K_T values	-1E-07	0.841457	1.281729	1.751077	2.054189	2.326785

E. **IDF Model Calibration**

Sherman's IDF model is given as (4)

$$I = \frac{CT_r^m}{T_d^e} \quad (4)$$

I = rainfall intensity, C, m and a = model parameters, Tr = return period (year) and Td = duration (hours)

Excel optimization solver was used to calibrate the non-linear power law given by Equation (4) to obtain the model parameters C, m and a.

F. **Goodness of Fit Test**

Using the Anderson-Darling test, the Normal distribution and Pearson Type 3 fit the rainfall intensities with 0.7115 and 0.7538 significant values at the 5% confidence level, respectively.

III. **Results and Discussion**

The Normal distribution model best fits the highest rainfall amounts as indicated in Table 3 according to the values for the estimated coefficient of determination, R^2, and Mean Square Error for a particular return period.

The values for rainfall intensity were computed using Equation (1). The mean and standard deviation of the Normal distribution of rainfall intensity are seen in Table 1. Equation (2) is used to compute the probability equivalent of rainfall intensity using Normal distribution for a duration of 10 minutes and a return period of 5 years by substituting the values of X_T, K_T, and S from Table 1.

Using the Anderson-Darling test, the Normal distribution and Pearson Type 3 fit the rainfall intensities with 0.7115 and 0.7538 significant values at the 5% confidence level, respectively.

X_T = 116.3 + (0.841x 51.8)) = 159.86 mm/hr

A. Specified Return Period IDF Models Calibration

The calibration of Sherman equation IDF models for given return durations was shown in (David et al., 2019). The results for the Normal distribution are shown in Table 3, together with the mean square error (MSE) and R^2 coefficients of determination.

Model parameters C, m, and a, are generated for a particular duration and return period after Equation (4) is calibrated using Excel optimization software. These IDF models are return period specific, as opposed to the non-specific models that are shown in Table 3. (see Equation 7).

Table 3: Developed IDF Models for different return periods using Normal Distribution rainfall intensity values for Abeokuta

Return Period	**IDF Model ±**	**Coefficient of Determination (R^2)**	**Mean Squared Error (MSE)**
2	$\mathrm{I} = \frac{4.898 {T_r}^{6.764}}{{T_d}^{0.543}}$	0.977	85.73

5	$$I = \frac{2.251 T_r^{\ 3.603}}{T_d^{\ 0.573}}$$	0.985	94.57
10	$$I = \frac{1.695 T_r^{\ 2.703}}{T_d^{\ 0.584}}$$	0.988	100.59
25	$$I = \frac{1.312 T_r^{\ 2.054}}{T_d^{\ 0.594}}$$	0.980	107.93
50	$$I = \frac{1.193 T_r^{\ 1.734}}{T_d^{\ 0.600}}$$	0.991	113.15
100	$$I = \frac{1.107 T_r^{\ 1.504}}{T_d^{\ 0.604}}$$	0.991	118.14

± return period specific IDF models

B. Evaluation of iterative Equation Solver in Excel

The Excel Solver software was used to evaluate the model's parameters over a 100-year defined return period. The generic IDF model presented in Equation has undergone ten (10) iterations (7).

Table 4: Model parameters for Sherman's specific IDF model calibration

Iteration	C	m	a
1	1	1	1
2	1.08686	1.4	0.81552
3	1.11005	1. 54	0.727277
4	1.132479	1.635577	0.74341
5	1. 26015	1.45596	0. 5802
6	1.3352	1.46344	0.61191
7	1. 27792	1.46665	0.59554
8	1. 3161	1.46593	0.60365
9	1. 31674	1.46587	0.60393
10	1. 31671	1.46587	0.60392
11	1. 31671	1.46587	0.60392
12	1. 31671	1.46587	0.60392

Table 5: Tabular Computation of Coefficient of Determination for Abeokuta

Intensity, I	Predicted Intensity, I_p	$(I-I_p)^2$	$(I-I_{avg})^2$
408.38	425.71	300.28	127092.25

298.33	280.10	332.31	60737.60
229.75	219.26	109.94	31637.74
195.95	184.30	135.82	20756.16
149.11	144.27	23.44	9453.67
114.79	112.93	3.44	3957.67
92.75	94.92	4.72	1670.36
66.69	74.31	58.01	219.34
51.88	62.46	111.85	196.28
37.87	48.89	121.46	43.16
31.30	41.09	95.92	35.64
25.33	35.91	112.00	53.00
18.05	29.31	126.76	13057.98
132.32		1535.96	268910.86

A General IDF model was also developed. A total of 13 durations multiplied by 6 return period yields 78 input data point. The entire input data were taken from Table 1.

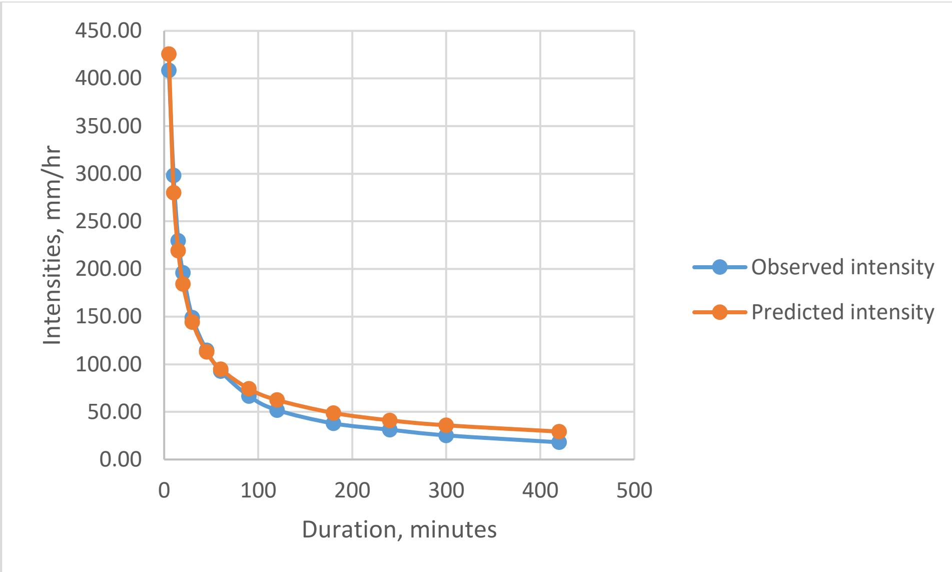
Programmed least squares equations were used to develop a general (non-specified) IDF model using Excel Optimization Solver. This resulted in Equation (7).

$$I = \frac{330.98 {T_r}^{0.135}}{{T_d}^{0.592}} \qquad (7)$$

Coefficient of determinant (R^2) = 0.989; Mean Squared Error = 29.23 mm/hr

A. Comparison of Observed and Predicted Rainfall Intensities

This model enables one to predict the intensity of rainfall of any duration and any return period. The verification of the developed model is carried out by plotting the observed and predicted intensities on the same log-log graph as shown in Figures 2.



Figures 2: Comparative plot for observed and predicted intensities

B. Comparison of Regression Approach and Excel Optimization Solver results for model parameters, R^2 and MSE

Table 6 (an extension of Table 5) clearly shows the result from Excel Optimization Solver option is more reliable than the normal regression method, the conventional simultaneous solution using matrix i.e. Gauss elimination, inverse or determinant approach.

Table 6: Results from Regression Approach and Excel Solver Optimization Approach (Normal distribution, 100-year return period)

Method	C	m	A	R^2	MSE
Regression	65.42	3.533	0.575	0.865	324.40
Excel solver	1. 317	1.466	0.604	0.991	118.14

I. Conclusion

The trend of higher intensities occurring at lower duration which is found in literature has been observed in the developed models for Normal distribution and Pearson Type 3 distributions.

The prediction of rainfall intensity with the PDFs showed a good match with observed intensity values. The Normal distribution model ranked as the best with respect to MSE 118.14 and R^2 0.991 in the return period specific model. The developed models can be used to obtain design intensity for drainage design for effective flood control.

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